that different intensification factors are applied to  $\sigma_h$  and  $\sigma_s$ . For example, for a circular cylinder having a diameter ratio,  $r_o/r_i$ , of 2.5 containing a single elliptic side hole of axis ratio a/b equal to 2.0

$$(\sigma_{h})_{\max} = \frac{4 \rho_{o} R^{a}}{R^{2} - 1}$$
$$\sigma_{s} = \frac{p_{o} \pi R^{2} R_{s}}{(R - 1) [\pi R_{s} (R + 1) - 1.5]}$$

and

$$K = 2 - \frac{3.5\pi}{2\pi (3.5) - 1.5} = 1.46$$

As data are not available for multiple elliptical holes similar to those in Figure 6 for multiple circular holes, this case cannot be analyzed at this time.

## **Experimental Methods**

Two experimental techniques, strain gages and photoelasticity, have been used to determine stress concentrations in cylinders containing side holes or pipe branches. The latter technique has received somewhat more attention than the former; however, so few data have been reported on this problem that the merits of each cannot be evaluated. Naturally, photoelasticity is of great value in solving complex problems, especially where the principal stress directions are not known. On the other hand, the use of strain gages, where applicable, saves considerable time and does not require the specialized knowledge needed for photoelastic work. In this discussion data derived from these two experimental techniques are reported.

Photoelastic Tests. Photoelastic tests, using the frozen stress technique, were performed on cylinders containing circular side holes; only final results are reported here. Two experimental stress concentration measurements were made using circular cylinders having a diameter ratio,  $r_o/r_i$ , of 2.5; one cylinder contained a single circular side hole having an  $R_s$  of 2.0 and the other cylinder contained a cross-bore circular hole having an  $R_{s}$  of 2.0. One photoelastic test was conducted on a cylinder having an R value of 2.5 and containing a crossbore slot. The cross-bore slot was made by drilling two circular holes tangent to each other and milling out the central portion, so that the long axis of the slot was perpendicular to the longitudinal axis of the cylinder. The ratio of length to width of the slot was 2.0this geometry of side hole is thus intermediate between a circular side hole and an elliptic side hole.

Strain Gage Tests. For convenience in mounting strain gages and to eliminate the necessity of making strain measurements under high fluid pressure (as it would be in a metal cylinder), a plastic

material was used for the cylinders: An epoxy resin (Ciba Co.'s Araldite 502). The cylinders were cast initially into semicylinders, leaving the bore exposed for easy access. The semicylinders were clamped together to form a cylinder, and circular cross-bore side holes drilled. Initially, two cylinders were fabricated. One cylinder (modulus of elasticity of 355,500 pounds per square inch and Poisson ratio of 0.44) had an R value of 2.0 and an  $R_s$  value of 1.0. The other cylinder (modulus of elasticity of 473,000 pounds per square inch and Poisson ratio of 0.37) had an R value of 4.0 and an  $R_s$  value of 2.0. This latter cylinder was subsequently reduced, by turning down the outside diameter, to cylinders having R values of 3, 2.5, 2.0, and 1.5; in all cases  $R_s$  remained at a constant value of 2.0. Thus one cylinder provided for five tests.

After the circular side holes were bored, the cylinder halves were separated and SR-4 electrical resistance gages (Type 8A-3, 60-ohm, 1/8-inch gage length) were mounted on the main cylinder bore near the side-hole interface, approximately as sketched on Figure 7, by using Type Al Epon straingage cement. Five hoop-direction and five longitudinal-direction gages mounted within 1/2 inch from the side hole in each half of the cylinder provided data for stress-concentration calculations. Gages were also mounted 4 to 6 inches from the side hole, where it was felt that stress-concentration effects would be small.

After the gages were cemented in place, the two cylinder halves were cemented together, plugs of the cylinder material were cast into the ends of the cylinder, and provision was made in the



Figure 7. Location of strain gages on bore of cylinder

Table II.	Data	Used in	Calculating	Stress	Concentrations	for	Cylinder	of	Wall
	18	Ratio R	= 2  and  S	ide Hol	e Ratio $R_{\rm e} = 1$		( î		

Distance from	Strains, I	nch/Inch <sup>a</sup>	Hoop Stress			
Side Hole, Inches	Hoop	Longitudinal	Conen. Factor, K			
0	+0.000833	ь	3.020			
0.10	+0.000725	-0.000190	2.820			
0.30	+0.000600	-0.000105	2.462			
0.50	+0.000528	-0.000065	2.187			
0.70	+0.000471	-0.000040	1.927			
1.00	+0.000415	-0.000020	1.677			
2.00	+0.000310	+0.000020	1.217			

 $^a$  For this cylinder E=355,500 lb./sq. inch.,  $\mu=0.44,$  o.d. = 3.875 inches, i.d. = 1.9375 inches.

<sup>b</sup> At side hole interface  $\sigma_r = \sigma_z = -p$ ; this information leads to a calculated value of K at interface of 3.020.